Formula Sheet for Energy in waves by GRIT Classes (9836442104)

For $(AK \ll 1)$:

1) $$\frac{dK}{dx} = \frac{1}{2}\mu_0\left(\frac{\partial y}{\partial t}\right)^2$$

2) $$\frac{dU}{dx} = \frac{1}{2}T\left(\frac{\partial y}{\partial x}\right)^2$$

3) $$\frac{dE}{dx} = \frac{dK}{dx} + \frac{dU}{dx}$$

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For Progressive wave:

4) Transverse

$$V = \sqrt{\frac{T}{\mu_0}} \qquad \boxed{T = V^2\mu_0}$$

Longitudinal

$$V = \sqrt{\frac{Y}{\rho_0}} \qquad \boxed{Y = V^2\rho_0}$$

5) $$\boxed{\mu_0 = \rho_0 S}$$

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For Transverse wave:

6) $$\frac{dE}{dx} = \frac{1}{2}\mu_0\underbrace{\left[\left(\frac{\partial y}{\partial t}\right)^2 + V^2\left(\frac{\partial y}{\partial x}\right)^2\right]}_{\text{say it } G}$$

7) $$G = \left(\frac{\partial y}{\partial t}\right)^2 + V^2\left(\frac{\partial y}{\partial x}\right)^2$$

8) $$\frac{dE}{dx} = \frac{1}{2}\mu_0 G$$

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9) $$\frac{\text{Energy}}{\text{Volume}} = \frac{1}{2}\rho_0 G$$

10) For Travelling wave $[\xi(x,t) = f(x \pm Vt)]$

$$\frac{\partial y}{\partial t} = V\left(\frac{\partial y}{\partial x}\right)$$

$$\therefore \frac{dK}{dx} = \frac{dU}{dx} = \frac{1}{2}\left(\frac{dE}{dx}\right)$$

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For Sinusoidal Travelling wave:

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(1) KE in betw^n $x=0$ to $x=\lambda$: $KE = \frac{\mu_0 A^2 \omega^2 \lambda}{4}$

$\Rightarrow$ KE " " $x=0$ to $x=n\lambda$: $KE = \frac{\mu_0 A^2 \omega^2}{4}(n\lambda)$

$\Rightarrow$ PE " " $x=0$ to $x=\lambda$: $KE = \frac{\mu_0 A^2 \omega^2}{4}\lambda$

$\Rightarrow$ PE " " $x=0$ to $x=\underline{n\lambda}$: $PE = \frac{\mu_0 A^2 \omega^2}{4}(n\lambda)$

$\rightarrow$ same as K.E

$\Rightarrow$ Total Energy " " $x=0$ to $x=\lambda$: $TE = \frac{\mu_0 A^2 \omega^2}{2}\lambda$

$\Rightarrow$ " " " " $x=0$ to $x=n\lambda$: $TE = \frac{\mu_0 A^2 \omega^2}{2}(n\lambda)$

$= 2(KE)$

$= 2(PE)$

Note: $K.E = \frac{\mu_0 A^2 \omega^2}{4}(l)$ will be valid only if length (l) is integral multiple of $\frac{\lambda}{4}$

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12) Avg. Energy per unit length $= \frac{1}{2}\mu A^2\omega^2$

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13) Avg. Energy flow per unit time $= \frac{1}{2}\mu A^2\omega^2 V$

14) $P = F_y \cdot V_p = (T_H \cdot \tan\theta)\left(\frac{\partial\xi}{\partial t}\right) = T_H\left(\frac{\partial y}{\partial x}\right)\left(\frac{\partial \xi}{\partial t}\right)$

15) $P_{avg} = \frac{1}{2}\mu A^2\omega^2 V =$ Avg. Energy flow per unit time

16) $I = \frac{P_{avg}}{A_\perp} = \frac{P_{avg}}{S} = \frac{1}{2}\rho A^2\omega^2 V =$ Avg. Energy flow per unit area per unit time.